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Spaces X in which all prime z -ideals of $C(X)$ are minimal or maximal

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Abstract: Quasi P -spaces are defined to be those Tychonoff spaces X such that each prime z -ideal of $C(X)$ is either minimal or maximal. This article is devoted to a systematic study of these spaces, which are an obvious generalization of P -spaces. The compact quasi P -spaces are characterized as the compact spaces which are scattered and of Cantor-Bendixson index no greater than 2. A thorough account of locally compact quasi P -spaces is given. If X is a cozero-complemented space and every nowhere dense zero set is a z -embedded P -space, then X is a quasi P -space. Conversely, if X is a quasi P -space and F is a nowhere dense z -embedded zero set, then F is a P -space. On the other hand, there are examples of countable quasi P -spaces with no P -points at all. If a product $X \times Y$ is normal and quasi P , then one of the factors must be a P -space. Conversely, if one of the factors is a compact quasi P -space and the other a P -space then the product is quasi P . If X is normal and X and Y are cozero-complemented spaces and $f : X \rightarrow Y$ is a closed continuous surjection which has the property that $f^{-1}(Z)$ is nowhere dense for each nowhere dense zero set Z , then if X is quasi P , so is Y . The converse fails even with more stringent assumptions on the map f . The paper then closes with a number of open questions, amongst which the most glaring is whether the free union of quasi P -spaces is always quasi P .

Keywords: quasi P -space, P -space, scattered space, Cantor-Bendixson derivatives,

nodec space, quasinormality

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