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Perfect mappings in topological groups, cross-complementary subsets and quotients

Abstract: The following general question is considered. Suppose that G is a topological group, and F, M are subspaces of G such that $G = MF$. Under these general assumptions, how are the properties of F and M related to the properties of G? For example, it is observed that if M is closed metrizable and F is compact, then G is a paracompact p-space. Furthermore, if M is closed and first countable, F is a first countable compactum, and $FM = G$, then G is also metrizable. Several other results of this kind are obtained. An extensive use is made of the following old theorem of N. Bourbaki [5]: if F is a compact subset of a topological group G, then the natural mapping of the product space $G \times F$ onto G, given by the product operation in G, is perfect (that is, closed continuous and the fibers are compact). This fact provides a basis for applications of the theory of perfect mappings to topological groups. Bourbaki's result is also generalized to the case of Lindelöf subspaces of topological groups; with this purpose the notion of a G_δ-closed mapping is introduced. This leads to new results on topological groups which are P-spaces.