Luisa Fattorusso Differentiability of weak solutions of nonlinear second order parabolic systems with quadratic growth and nonlinearity $q \ge 2$

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Abstract: Let Ω be a bounded open subset of \mathbb{R}^n , let X = (x, t) be a point of $\mathbb{R}^n \times \mathbb{R}^N$. In the cylinder $Q = \Omega \times (-T, 0)$, T > 0, we deduce the local differentiability result

$$u\in L^2(-a,0,H^2(B(\sigma),\mathbb{R}^N))\cap H^1(-a,0,L^2(B(\sigma),\mathbb{R}^N))$$

for the solutions u of the class $L^q(-T, 0, H^{1,q}(\Omega, \mathbb{R}^N)) \cap C^{0,\lambda}(\overline{Q}, \mathbb{R}^N)$ $(0 < \lambda < 1, N \text{ integer } \geq 1)$ of the nonlinear parabolic system

$$-\sum_{i=1}^{n} D_{i}a^{i}(X, u, Du) + \frac{\partial u}{\partial t} = B^{0}(X, u, Du)$$

with quadratic growth and nonlinearity $q \geq 2$. This result had been obtained making use of the interpolation theory and an imbedding theorem of Gagliardo-Nirenberg type for functions u belonging to $W^{1,q} \cap C^{0,\lambda}$.

Keywords: differentiability of weak solution, parabolic systems, nonlinearity with q > 2

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