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Essential P-spaces: a generalization of door spaces

Abstract: An element f of a commutative ring A with identity element is called a von Neumann regular element if there is a g in A such that $f^2 g = f$. A point p of a (Tychonoff) space X is called a P-point if each f in the ring $C(X)$ of continuous real-valued functions is constant on a neighborhood of p. It is well-known that the ring $C(X)$ is von Neumann regular ring iff each of its elements is a von Neumann regular element; in which case X is called a P-space. If all but at most one point of X is a P-point, then X is called an essential P-space. In earlier work it was shown that X is an essential P-space iff for each f in $C(X)$, either f or $1 - f$ is von Neumann regular element. Properties of essential P-spaces (which are generalizations of J.L. Kelley's door spaces) are derived with the help of the algebraic properties of $C(X)$. Despite its simple sounding description, an essential P-space is not simple to describe definitively unless its non P-point η is a G_δ, and not even then if there are infinitely many pairwise disjoint cozerosets with η in their closure. The general case is considered and open problems are posed.

Keywords: P-point, P-space, essential P-space, door space, F-space, basically disconnected space, space of minimal prime ideals, SV-ring, SV-space, rank, von Neumann regular ring, von Neumann local ring, Lindelöf space
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