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***Properties of one-point completions of a noncompact metrizable space***

Comment.Math.Univ.Carolinae 46,1 (2005) 105-123.

**Abstract:** If a metrizable space $X$ is dense in a metrizable space $Y$, then $Y$ is called a metric extension of $X$. If $T_1$ and $T_2$ are metric extensions of $X$ and there is a continuous map of $T_2$ into $T_1$ keeping $X$ pointwise fixed, we write $T_1 \leq T_2$. If $X$ is noncompact and metrizable, then $(\mathcal{M}(X), \leq)$ denotes the set of metric extensions of $X$, where $T_1$ and $T_2$ are identified if $T_1 \leq T_2$ and $T_2 \leq T_1$, i.e., if there is a homeomorphism of $T_1$ onto $T_2$ keeping $X$ pointwise fixed. $(\mathcal{M}(X), \leq)$ is a large complicated poset studied extensively by V. Bel'nov [The structure of the set of metric extensions of a noncompact metrizable space, Trans. Moscow Math. Soc. 32 (1975), 1–30]. We study the poset $(\mathcal{E}(X), \leq)$ of one-point metric extensions of a locally compact metrizable space $X$. Each such extension is a (Cauchy) completion of $X$ with respect to a compatible metric. This poset resembles the lattice of compactifications of a locally compact space if $X$ is also separable. For Tychonoff $X$, let $X^* = \beta X \backslash X$, and let $\mathcal{Z}(X)$ be the poset of zerosets of $X$ partially ordered by set inclusion.
Theorem If $X$ and $Y$ are locally compact separable metrizable spaces, then $(\mathcal{E}(X), \leq)$ and $(\mathcal{E}(Y), \leq)$ are order-isomorphic iff $\mathcal{Z}(X^*)$ and $\mathcal{Z}(Y^*)$ are order-isomorphic, and iff $X^*$ and $Y^*$ are homeomorphic. We construct an order preserving bijection $\lambda : \mathcal{E}(X) \rightarrow \mathcal{Z}(X^*)$ such that a one-point completion in $\mathcal{E}(X)$ is locally compact iff its image under $\lambda$ is clopen. We extend some results to the nonseparable case, but leave problems open. In a concluding section, we show how to construct one-point completions geometrically in some explicit cases.

**Keywords:** metrizable, metric extensions and completions, completely metrizable, one-point metric extensions, extension traces, zerosets, clopen sets, Stone-Čech compactification, $\beta X \backslash X$, hedgehog
**AMS Subject Classification:** Primary 54E45, 54E50; Secondary 54E35, 54D35