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*Duality theory of spaces of vector-valued continuous functions*

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**Abstract:** Let  $X$  be a completely regular Hausdorff space,  $E$  a real normed space, and let  $C_b(X, E)$  be the space of all bounded continuous  $E$ -valued functions on  $X$ . We develop the general duality theory of the space  $C_b(X, E)$  endowed with locally solid topologies; in particular with the strict topologies  $\beta_z(X, E)$  for  $z = \sigma, \tau, t$ . As an application, we consider criteria for relative weak-star compactness in the spaces of vector measures  $M_z(X, E')$  for  $z = \sigma, \tau, t$ . It is shown that if a subset  $H$  of  $M_z(X, E')$  is relatively  $\sigma(M_z(X, E'), C_b(X, E))$ -compact, then the set  $\text{conv}(S(H))$  is still relatively  $\sigma(M_z(X, E'), C_b(X, E))$ -compact ( $S(H)$  = the solid hull of  $H$  in  $M_z(X, E')$ ). A Mackey-Arens type theorem for locally convex-solid topologies on  $C_b(X, E)$  is obtained.

**Keywords:** vector-valued continuous functions, strict topologies, locally solid topologies, weak-star compactness, vector measures

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