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On the range of a closed operator in an L_1-space of vector-valued functions

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Abstract: Let X be a reflexive Banach space and A be a closed operator in an L_1-space of X-valued functions. Then we characterize the range $R(A)$ of A as follows. Let $0 \neq \lambda_n \in \rho(A)$ for all $1 \leq n < \infty$, where $\rho(A)$ denotes the resolvent set of A, and assume that $\lim_{n\to\infty} \lambda_n = 0$ and $\sup_{n\geq 1} \|\lambda_n(\lambda_n - A)^{-1}\| < \infty$. Furthermore, assume that there exists $\lambda_\infty \in \rho(A)$ such that $\|\lambda_\infty(\lambda_\infty - A)^{-1}\| \leq 1$. Then $f \in R(A)$ is equivalent to $\sup_{n\geq 1} \|(\lambda_n - A)^{-1} f\|_1 < \infty$. This generalizes Shaw's result for scalar-valued functions.

Keywords: reflexive Banach space, L_1-space of vector-valued functions, closed operator, resolvent set, range and domain, linear contraction, C_0-semigroup, strongly continuous cosine family of operators
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