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Dimension in algebraic frames, II: Applications to frames of ideals in $C(X)$

Comment.Math.Univ.Carolinae 46,4 (2005) 607-636.

Abstract: This paper continues the investigation into Krull-style dimensions in algebraic frames.
Let L be an algebraic frame. $dim(L)$ is the supremum of the lengths k of sequences $p_0 < p_1 < \cdots < p_k$ of (proper) prime elements of L. Recently, Th. Coquand, H. Lombardi and M.-F. Roy have formulated a characterization which describes the dimension of L in terms of the dimensions of certain boundary quotients of L. This paper gives a purely frame-theoretic proof of this result, at once generalizing it to frames which are not necessarily compact. This result applies to the frame $\mathcal{C}_z(X)$ of all z-ideals of $C(X)$, provided the underlying Tychonoff space X is Lindelöf. If the space X is compact, then it is shown that the dimension of $\mathcal{C}_z(X)$ is at most n if and only if X is scattered of Cantor-Bendixson index at most $n+1$.
If X is the topological union of spaces X_i, then the dimension of $\mathcal{C}_z(X)$ is the supremum of the dimensions of the $\mathcal{C}_z(X_i)$. This and other results apply to the frame of all d-ideals $\mathcal{C}_d(X)$ of $C(X)$, however, not the characterization in terms of boundaries. An explanation of this is given within, thus marking some of the differences between these two frames and their dimensions.

Keywords: dimension of a frame, z-ideals, scattered space, natural typing of open sets
AMS Subject Classification: Primary 06D22, 54C30; Secondary 03G10, 16P60, 54B35