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Topological structure of the space of lower semi-continuous functions

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Abstract: Let $L(X)$ be the space of all lower semi-continuous extended real-valued functions on a Hausdorff space X, where, by identifying each f with the epi-graph $epi(f)$, $L(X)$ is regarded the subspace of the space $Cld_F^*(X \times \mathbb{R})$ of all closed sets in $X \times \mathbb{R}$ with the Fell topology. Let

$LSC(X) = \{f \in L(X) \mid f(X) \cap \mathbb{R} \neq \emptyset, f(X) \subset (-\infty, \infty]\}$ and$LSC_B(X) = \{f \in L(X) \mid f(X)$ is a bounded subset of $\mathbb{R}\}$.

We show that $L(X)$ is homeomorphic to the Hilbert cube $Q = [-1,1]^{\mathbb{N}}$ if and only if X is second countable, locally compact and infinite. In this case, it is proved that $(L(X), LSC(X), LSC_B(X))$ is homeomorphic to $(ConeQ, Q \times (0,1), \Sigma \times (0,1))$ (resp. (Q, s, Σ)) if X is compact (resp. X is non-compact), where $ConeQ = (Q \times \mathbf{I})/(Q \times \{1\})$ is the cone over Q, $s = (-1,1)^{\mathbb{N}}$ is the pseudo-interior, $\Sigma = \{(x_i)_{i \in \mathbb{N}} \in Q \mid \sup_{i \in \mathbb{N}} |x_i| < 1\}$ is the radial-interior.

Keywords: space of lower semi-continuous functions, epi-graph, Fell topology, Hilbert cube, pseudo-interior, radial-interior
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