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Topologies generated by ideals

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Abstract: A topological space X is said to be generated by an ideal $\mathcal{I}$ if for all $A \subseteq X$ and all $x \in \overline{A}$ there is $E \subseteq A$ in $\mathcal{I}$ such that $x \in \overline{E}$, and is said to be weakly generated by $\mathcal{I}$ if whenever a subset A of X contains $\overline{E}$ for every $E \subseteq A$ with $E \in \mathcal{I}$, then A itself is closed. An important class of examples are the so called weakly discretely generated spaces (which include sequential, scattered and compact Hausdorff spaces). Another paradigmatic example is the class of Alexandroff spaces which corresponds to spaces generated by finite sets. By considering an appropriate topology on the power set of X we show that τ is weakly generated by $\mathcal{I}$ iff τ is a $\mathcal{I}$-closed subset of $\mathcal{P}(X)$. The class of spaces weakly generated by an ideal behaves as the class of sequential spaces, in the sense that their closure operator can be characterized as the sequential closure and moreover there is a natural notion of a convergence associated to them. We also show that the collection of topologies weakly generated by $\mathcal{I}$ is lattice isomorphic to a lattice of pre-orders over $\mathcal{I}$.

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