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More on cardinal invariants of analytic P-ideals

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Abstract: Given an ideal $\mathcal{I}$ on ω let $\mathfrak{a}(\mathcal{I})$ ($\bar{\mathfrak{a}}(\mathcal{I})$) be minimum of the cardinalities of infinite (uncountable) maximal $\mathcal{I}$-almost disjoint subsets of $[\omega]^\omega$. We show that $\mathfrak{a}(\mathcal{I}_h) > \omega$ if $\mathcal{I}_h$ is a summable ideal; but $\mathfrak{a}(\mathcal{Z}_{\vec{\mu}}) = \omega$ for any tall density ideal $\mathcal{Z}_{\vec{\mu}}$ including the density zero ideal $\mathcal{Z}$. On the other hand, you have $\mathfrak{b} \le \bar{\mathfrak{a}}(\mathcal{I})$ for any analytic P-ideal $\mathcal{I}$, and $\bar{\mathfrak{a}}(\mathcal{Z}_{\vec{\mu}}) \le \mathfrak{a}$ for each density ideal $\mathcal{Z}_{\vec{\mu}}$. For each ideal $\mathcal{I}$ on ω denote $\mathfrak{b}_{\mathcal{I}}$ and $\mathfrak{d}_{\mathcal{I}}$ the unbounding and dominating numbers of $\langle \omega^\omega, \le_{\mathcal{I}} \rangle$ where $f \le_{\mathcal{I}} g$ iff $\{n \in \omega : f(n) > g(n)\} \in \mathcal{I}$. We show that $\mathfrak{b}_{\mathcal{I}} = \mathfrak{b}$ and $\mathfrak{d}_{\mathcal{I}} = \mathfrak{d}$ for each analytic P-ideal $\mathcal{I}$. Given a Borel ideal $\mathcal{I}$ on ω we say that a poset $\mathbb{P}$ is *$\mathcal{I}$-bounding* iff $\forall x \in \mathcal{I} \cap V^{\mathbb{P}}\ \exists y \in \mathcal{I} \cap V\ x \subseteq y$. $\mathbb{P}$ is *$\mathcal{I}$-dominating* iff $\exists y \in \mathcal{I} \cap V^{\mathbb{P}}\ \forall x \in \mathcal{I} \cap V\ x \subseteq^* y$. For each analytic P-ideal $\mathcal{I}$ if a poset $\mathbb{P}$ has the Sacks property then $\mathbb{P}$ is $\mathcal{I}$-bounding; moreover if $\mathcal{I}$ is tall as well then the property $\mathcal{I}$-bounding/$\mathcal{I}$-dominating implies ω^ω-bounding/adding dominating reals, and the converses of these two implications are false. For the density zero ideal $\mathcal{Z}$ we can prove more: (i) a poset $\mathbb{P}$ is $\mathcal{Z}$-bounding iff it has the Sacks property, (ii) if $\mathbb{P}$ adds a slalom capturing all ground model reals then $\mathbb{P}$ is $\mathcal{Z}$-dominating.

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