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Interpolation of κ-compactness and PCF

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Abstract: We call a topological space κ-compact if every subset of size κ has a complete accumulation point in it. Let $\Phi(\mu,\kappa,\lambda)$ denote the following statement: $\mu < \kappa < \lambda = \mathrm{cf}(\lambda)$ and there is $\{S_\xi : \xi < \lambda\} \subset [\kappa]^\mu$ such that $|\{\xi : |S_\xi \cap A| = \mu\}| < \lambda$ whenever $A \in [\kappa]^{<\kappa}$. We show that if $\Phi(\mu,\kappa,\lambda)$ holds and the space X is both μ-compact and λ-compact then X is κ-compact as well. Moreover, from PCF theory we deduce $\Phi(\mathrm{cf}(\kappa),\kappa,\kappa^+)$ for every singular cardinal κ. As a corollary we get that a linearly Lindelöf and $\aleph_\omega$-compact space is uncountably compact, that is κ-compact for all uncountable cardinals κ.

Keywords: complete accumulation point, κ-compact space, linearly Lindelöf space, PCF theory

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