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*Weyl quantization for the semidirect product of a compact Lie group and a vector space*

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**Abstract:** Let  $G$  be the semidirect product  $V \rtimes K$  where  $K$  is a semisimple compact connected Lie group acting linearly on a finite-dimensional real vector space  $V$ . Let  $\mathcal{O}$  be a coadjoint orbit of  $G$  associated by the Kirillov-Kostant method of orbits with a unitary irreducible representation  $\pi$  of  $G$ . We consider the case when the corresponding little group  $H$  is the centralizer of a torus of  $K$ . By dequantizing a suitable realization of  $\pi$  on a Hilbert space of functions on  $\mathbb{C}^n$  where  $n = \dim(K/H)$ , we construct a symplectomorphism between a dense open subset of  $\mathcal{O}$  and the symplectic product  $\mathbb{C}^{2n} \times \mathcal{O}'$  where  $\mathcal{O}'$  is a coadjoint orbit of  $H$ . This allows us to obtain a Weyl correspondence on  $\mathcal{O}$  which is adapted to the representation  $\pi$  in the sense of [B. Cahen, *Quantification d'une orbite massive d'un groupe de Poincaré généralisé*, C.R. Acad. Sci. Paris t. 325, série I (1997), 803–806].

**Keywords:** Weyl quantization, Berezin quantization, semidirect product, coadjoint orbits, unitary representations

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