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Almost disjoint families and "never" cardinal invariants

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Abstract: We define two cardinal invariants of the continuum which arise naturally from combinatorially and topologically appealing properties of almost disjoint families of sets of the natural numbers. These are the *never soft* and *never countably paracompact* numbers. We show that these cardinals must both be equal to ω_1 under the effective weak diamond principle $\diamondsuit(\omega, \omega, <)$, answering questions of da Silva S.G., *On the presence of countable paracompactness, normality and property* (a) *in spaces from almost disjoint families*, Questions Answers Gen. Topology **25** (2007), no. 1, 1–18, and give some information about the strength of this principle.

Keywords: almost disjoint families, parametrized weak diamond principles, property (a), countable paracompactness
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