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## *FC-modules with an application to cotorsion pairs*

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**Abstract:** Let  $R$  be a ring. A left  $R$ -module  $M$  is called an FC-module if  $M^+ = \text{Hom}_{\mathbb{Z}}(M, \mathbb{Q}/\mathbb{Z})$  is a flat right  $R$ -module. In this paper, some homological properties of FC-modules are given. Let  $n$  be a nonnegative integer and  $\mathcal{FC}_n$  the class of all left  $R$ -modules  $M$  such that the flat dimension of  $M^+$  is less than or equal to  $n$ . It is shown that  $({}^{\perp}(\mathcal{FC}_n^{\perp}), \mathcal{FC}_n^{\perp})$  is a complete cotorsion pair and if  $R$  is a ring such that  $\text{fd}(({}_R R)^+) \leq n$  and  $\mathcal{FC}_n$  is closed under direct sums, then  $(\mathcal{FC}_n, \mathcal{FC}_n^{\perp})$  is a perfect cotorsion pair. In particular, some known results are obtained as corollaries.

**Keywords:** character modules, flat modules, cotorsion pairs

**AMS Subject Classification:** 16D40, 16D80, 16E99

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