Gennaro Infante, Paolamaria Pietramala

Perturbed Hammerstein integral inclusions with solutions that change sign

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Abstract: We establish new existence results for nontrivial solutions of some integral inclusions of Hammerstein type, that are perturbed with an affine functional. In order to use a theory of fixed point index for multivalued mappings, we work in a cone of continuous functions that are positive on a suitable subinterval of $[0, 1]$. We also discuss the optimality of some constants that occur in our theory. We improve, complement and extend previous results in the literature.

Keywords: fixed point index, cone, nontrivial solution
AMS Subject Classification: Primary 45G10; Secondary 34A60, 34B10, 47H04, 47H10, 47H30

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