

## Taras Banakh, Svetlana Dimitrova

### *Openly factorizable spaces and compact extensions of topological semigroups*

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**Abstract:** We prove that the semigroup operation of a topological semigroup  $S$  extends to a continuous semigroup operation on its Stone-Čech compactification  $\beta S$  provided  $S$  is a pseudocompact openly factorizable space, which means that each map  $f : S \rightarrow Y$  to a second countable space  $Y$  can be written as the composition  $f = g \circ p$  of an open map  $p : X \rightarrow Z$  onto a second countable space  $Z$  and a map  $g : Z \rightarrow Y$ . We present a spectral characterization of openly factorizable spaces and establish some properties of such spaces.

**Keywords:** topological semigroup, semigroup compactification, inverse spectrum, pseudocompact space, openly factorizable space, openly generated space, Eberlein compact, Corson compact, Valdivia compact

**AMS Subject Classification:** 22A15, 54B30, 54C20, 54C08, 54D35

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