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Uniqueness and non uniqueness of optimal maps in mass transport problem with not strictly convex cost

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Abstract: In the setting of the optimal transportation problem we provide some conditions which ensure the existence and the uniqueness of the optimal map in the case of cost functions satisfying mild regularity hypothesis and no convexity or concavity assumptions.

Keywords: mass transport problem, measurable selections, degree theory
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