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On reflexive closed set lattices

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Abstract: For a topological space X , let $S(X)$ denote the set of all closed subsets in X , and let $C(X)$ denote the set of all continuous maps $f : X \rightarrow X$. A family $\mathcal{A} \subseteq S(X)$ is called *reflexive* if there exists $\mathcal{C} \subseteq C(X)$ such that $\mathcal{A} = \{A \in S(X) : f(A) \subseteq A \text{ for every } f \in \mathcal{C}\}$. Every reflexive family of closed sets in space X forms a sub complete lattice of the lattice of all closed sets in X . In this paper, we continue to study the reflexive families of closed sets in various types of topological spaces. More necessary and sufficient conditions for certain families of closed sets to be reflexive are obtained.

Keywords: reflexive families of closed sets, closed set lattice, hyperspace, lower semicontinuous set-valued map

AMS Subject Classification: 54C05, 54C60, 54B20, 06B99

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