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*Nonassociativity in VOA theory and finite group theory*

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**Abstract:** We discuss some examples of nonassociative algebras which occur in VOA (vertex operator algebra) theory and finite group theory. Methods of VOA theory and finite group theory provide a lot of nonassociative algebras to study. Ideas from nonassociative algebra theory could be useful to group theorists and VOA theorists.

**Keywords:** nonassociative algebra, nonassociative commutative algebra, groups of Lie type, sporadic groups, vertex operator algebras, lattice type vertex operator algebras, axioms,  $(B, N)$ -pair, monster,  $2A$ -involutions, Jordan algebra, pairwise orthogonal idempotents,  $E_8$ ,  $E_6$ , polynomial identity

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