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Metric spaces with point character equal to their size

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Abstract: In this paper we consider the point character of metric spaces. This parameter which is a uniform version of dimension, was introduced in the context of uniform spaces in the late seventies by Jan Pelant, Cardinal reflections and point-character of uniformities, Seminar Uniform Spaces (Prague, 1973–1974), Math. Inst. Czech. Acad. Sci., Prague, 1975, pp. 149–158. Here we prove for each cardinal κ , the existence of a metric space of cardinality and point character κ . Since the point character can never exceed the cardinality of a metric space this gives the construction of metric spaces with "largest possible" point character. The existence of such spaces was already proved using GCH in Rödl V., Small spaces with large point character, European J. Combin. 8 (1987), no. 1, 55–58. The goal of this note is to remove this assumption.

Keywords: point character, uniform cover, continuum hypothesis, Specker graph. AMS Subject Classification: 05C12, 05C15, 54A99, 54A25, 03E05

References

- Avart C., Komjáth P., Luczak T., Rödl V., Colorful flowers, Topology Appl. 156 (2009), no. 7, 1386-1395.
- [2] Erdös P., Hajnal A., On chromatic number of infinite graphs, Theory of Graphs, P. Erdös and G. Katona, Eds., Akademiai Kiadó, Budapest, 1968, pp. 83–98.
- [3] Erdös P., Galvin F., Hajnal A., On set-systems having large chromatic number and not containing prescribed subsystems, Infinite and Finite Sets (A. Hajnal, R. Rado, V.T. Sós, Eds.), North Holland, 1976, pp. 425-513.
- [4] Stone A.H., Paracompactness and Product Spaces, Bull. Amer. Math. Soc. 54 (1948), 977–982.
- [5] Stone A.H., Universal Space for some Metrizable Uniformities, Quart. J. Math. 11 (1960), 105-115.
- [6] Isbell J.R., Uniform Spaces, Mathematical Surveys, 12, American Mathematical Society, Providence, RI, 1964.
- [7] J. Pelant, Cardinal reflections and point-character of uniformities, Seminar Uniform Spaces (Prague, 1973-1974), Math. Inst. Czech. Acad. Sci., Prague, 1975, pp. 149-158.
- [8] J. Pelant, Uniform metric spaces, Seminar Uniform Spaces 1975-1977 directed by Z. Frolík, Math. Inst. Czech. Acad. Sci., Prague, 1976, pp. 49-53.
- [9] Pelant J., Rödl V., On coverings of infinite-dimensional metric spaces, Discrete Math. 108 (1992), no. 1-3, 75-81.
- [10] Rödl V., Canonical partition relations and point character of ℓ₁ spaces, Seminar Uniform Spaces 1976-1977, pp. 79-81.
- $[11] \ \operatorname{R\"{o}dl} \operatorname{V.}, \textit{Small spaces with large point character}, \operatorname{European J. Combin. 8 (1987)}, \operatorname{no. 1}, 55-58.$
- [12] Schepin E.V., On a problem of Isbell, Dokl. Akad. Nauk SSSR 222 (1976), 541-543.