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Approximation and shape preserving properties of the nonlinear Bleimann-Butzer-Hahn operators of max-product kind

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Abstract: Starting from the study of the *Shepard nonlinear operator of max-prod type* in (Bede, Nobuhara et al., 2006, 2008), in the book (Gal, 2008), Open Problem 5.5.4, pp. 324–326, the *Bleimann-Butzer-Hahn max-prod type operator* is introduced and the question of the approximation order by this operator is raised. In this paper firstly we obtain an upper estimate of the approximation error of the form $\omega_1(f; (1+x)^{\frac{3}{2}}\sqrt{x/n})$. A consequence of this result is that for each compact subinterval $[0, a]$, with arbitrary $a > 0$, the order of uniform approximation by the Bleimann-Butzer-Hahn operator is less than $O(1/\sqrt{n})$. Then, one proves by a counterexample that in a sense, for arbitrary f this order of uniform approximation cannot be improved. Also, for some subclasses of functions, including for example the bounded, nondecreasing concave functions, the essentially better order $\omega_1(f; (x+1)^2/n)$ is obtained. Shape preserving properties are also investigated.

Keywords: nonlinear Bleimann-Butzer-Hahn operator of max-product kind, degree of approximation, shape preserving properties

AMS Subject Classification: 41A30, 41A25, 41A29

REFERENCES

- [1] Bede B., Gal S.G., *Approximation by nonlinear Bernstein and Favard-Szász-Mirakjan operators of max-product kind*, J. Concrete and Applicable Mathematics **8** (2010), no. 2, 193–207.
- [2] Bede B., Coroianu L., Gal S.G., *Approximation and shape preserving properties of the Bernstein operator of max-product kind*, Int. J. Math. Math. Sci. 2009, Art. ID 590589, 26 pp., doi:10.1155/2009/590589.
- [3] Bede B., Coroianu L., Gal S.G., *Approximation by truncated nonlinear Favard-Szász-Mirakjan operators of max-product kind*, Demonstratio Math., to appear.
- [4] Bede B., Nobuhara H., Fodor J., Hirota K., *Max-product Shepard approximation operators*, J. Advanced Computational Intelligence and Intelligent Informatics **10** (2006), 494–497.
- [5] Bede B., Nobuhara H., Daňková M., Di Nola A., *Approximation by pseudo-linear operators*, Fuzzy Sets and Systems **159** (2008), 804–820.
- [6] Bleimann G., Butzer P.L., Hahn L., *A Bernstein-type operator approximating continuous functions on the semi-axis*, Indag. Math. **42** (1980), 255–262.
- [7] Duman O., *Statistical convergence of max-product approximating operators*, Turkish J. Math. **33** (2009), 1–14.
- [8] Gal S.G., *Shape-Preserving Approximation by Real and Complex Polynomials*, Birkhäuser, Boston-Basel-Berlin, 2008.
- [9] Khan R.A., *A note on a Bernstein-type operator of Bleimann, Butzer and Hahn*, J. Approx. Theory **53** (1988), no. 3, 295–303.
- [10] Popoviciu T., *Deux remarques sur les fonctions convexes*, Bull. Soc. Sci. Acad. Roumaine **220** (1938), 45–49.