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Growth orders of Cesàro and Abel means of uniformly continuous operator semi-groups and cosine functions

Abstract: It will be proved that if N is a bounded nilpotent operator on a Banach space X of order $k + 1$, where $k \geq 1$ is an integer, then the γ-th order Cesàro mean $C_t^{\gamma} := \gamma t^{-\gamma} \int_0^t (t - s)^{\gamma-1} T(s)\, ds$ and Abel mean $A_{\lambda} := \lambda \int_0^{\infty} e^{-\lambda s} T(s)\, ds$ of the uniformly continuous semigroup $(T(t))_{t \geq 0}$ of bounded linear operators on X generated by $iaI + N$, where $0 \neq a \in \mathbb{R}$, satisfy that (a) $\|C_t^{\gamma}\| \sim t^{k-\gamma}$ $(t \to \infty)$ for all $0 < \gamma \leq k + 1$; (b) $\|C_t^{\gamma}\| \sim t^{-1}$ $(t \to \infty)$ for all $\gamma \geq k + 1$; (c) $\|A_{\lambda}\| \sim \lambda$ $(\lambda \downarrow 0)$. A similar result will be also proved for the uniformly continuous cosine function $(C(t))_{t \geq 0}$ of bounded linear operators on X generated by $(iaI + N)^2$.

References

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