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A priori estimates for quasilinear parabolic systems with quadratic nonlinearities in the gradient

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Abstract: We derive local a priori estimates of the Hölder norm of solutions to quasilinear elliptic systems with quadratic nonlinearities in the gradient. We assume higher integrability of solutions and smallness of its BMO norm but the Hölder norm is estimated in terms of BMO norm of the solution under consideration, only.

Keywords: quasilinear parabolic systems, quadratic nonlinearities, regularity, Morrey, VMO spaces

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