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Sharp generalized Trudinger inequalities via truncation for embedding into multiple exponential spaces

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Abstract: We prove that the generalized Trudinger inequality for Orlicz-Sobolev spaces embedded into multiple exponential spaces implies a version of an inequality due to Brézis and Wainger.

Keywords: Orlicz spaces, Sobolev inequalities

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