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A note on formal power series

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Abstract: In this note we investigate a relationship between the boundary behavior of power series and the composition of formal power series. In particular, we prove that the composition domain of a formal power series g is convex and balanced which implies that the subset $\overline{\mathbb{X}}_g$ consisting of formal power series which can be composed by a formal power series g possesses such properties. We also provide a necessary and sufficient condition for the superposition operator T_g to map $\overline{\mathbb{X}}_g$ into itself or to map $\mathbb{X}_g$ into itself, respectively.

Keywords: composition, end behavior of convergence of power series, convex and balanced set, formal power series
AMS Subject Classification: Primary 13F25; Secondary 40A30, 52A05

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