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Functional separability

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Abstract: A space X is functionally countable (FC) if for every continuous $f : X \to \mathbb{R}$, $|f(X)| \leq \omega$. The class of FC spaces includes ordinals, some trees, compact scattered spaces, Lindelöf P-spaces, σ-products in 2^κ, and some L-spaces. We consider the following three versions of functional separability: X is 1-FS if it has a dense FC subspace; X is 2-FS if there is a dense subspace $Y \subset X$ such that for every continuous $f : X \to \mathbb{R}$, $|f(Y)| \leq \omega$; X is 3-FS if for every continuous $f : X \to \mathbb{R}$, there is a dense subspace $Y \subset X$ such that $|f(Y)| \leq \omega$. We give examples distinguishing 1-FS, 2-FS, and 3-FS and discuss some properties of functionally separable spaces.

Keywords: functionally countable, pseudo-$\aleph_1$-compact, DCCC, P-space, τ-simple, scattered, 1-functionally separable, 2-functionally separable, 3-functionally separable, pseudocompact, dyadic compactum, σ-centered base, LOTS
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