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Steady compressible Navier–Stokes–Fourier system in two space dimensions

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Abstract: We study steady flow of a compressible heat conducting viscous fluid in a bounded two-dimensional domain, described by the Navier–Stokes–Fourier system. We assume that the pressure is given by the constitutive equation $p(\rho,\theta) \sim \rho^\gamma + \rho\theta$, where ρ is the density and θ is the temperature. For $\gamma > 2$, we prove existence of a weak solution to these equations without any assumption on the smallness of the data. The proof uses special approximation of the original problem, which guarantees the pointwise boundedness of the density. Thus we get a solution with density in $L^\infty(\Omega)$ and temperature and velocity in $W^{1,q}(\Omega)$ for any $q < \infty$.

Keywords: steady compressible Navier–Stokes–Fourier equations, slip boundary condition, weak solutions, large data
AMS Subject Classification: 35Q30, 76N10

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