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Isolated points and redundancy

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Abstract: We describe the isolated points of an arbitrary topological space (X, τ) . If the τ -specialization pre-order on X has enough maximal elements, then a point $x \in X$ is an isolated point in (X, τ) if and only if x is both an isolated point in the subspaces of τ -kerneled points of X and in the τ -closure of $\{x\}$ (a special case of this result is proved in Mehrvarz A.A., Samei K., *On commutative Gelfand rings*, J. Sci. Islam. Repub. Iran **10** (1999), no. 3, 193–196). This result is applied to an arbitrary subspace of the prime spectrum $\text{Spec}(R)$ of a (commutative with nonzero identity) ring R , and in particular, to the space $\text{Spec}(R)$ and the maximal and minimal spectrum of R . Dually, a prime ideal P of R is an isolated point in its Zariski-kernel if and only if P is a minimal prime ideal. Finally, some aspects about the redundancy of (maximal) prime ideals in the (Jacobson) prime radical of a ring are considered, and we characterize when $\text{Spec}(R)$ is a scattered space.

Keywords: maximal (minimal) spectrum of a ring, scattered space, isolated point, prime radical, Jacobson radical

AMS Subject Classification: 54F65, 13C05

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