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Boundedness of one-sided fractional integrals in the one-sided Calderón-Hardy spaces

Abstract: In this paper we study the mapping properties of the one-sided fractional integrals in the Calderón-Hardy spaces $\mathcal{H}_{q,\alpha}^{p,+}(\omega)$ for $0 < p \leq 1$, $0 < \alpha < \infty$ and $1 < q < \infty$. Specifically, we show that, for suitable values of p, q, γ, α and s, if $\omega \in A_s^+$ (Sawyer's classes of weights) then the one-sided fractional integral I_γ^+ can be extended to a bounded operator from $\mathcal{H}_{q,\alpha}^{p,+}(\omega)$ to $\mathcal{H}_{q,\alpha+\gamma}^{p,+}(\omega)$. The result is a consequence of the pointwise inequality

$$N_{q,\alpha+\gamma}^+ \left(I_\gamma^+ F; x \right) \leq C_{\alpha,\gamma} N_{q,\alpha}^+ \left(F; x \right),$$

where $N_{q,\alpha}^+(F; x)$ denotes the Calderón maximal function.

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