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Pcf theory and cardinal invariants of the reals

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Abstract: The *additivity spectrum* $\mathrm{ADD}(\mathcal{I})$ of an ideal $\mathcal{I} \subset \mathcal{P}(I)$ is the set of all regular cardinals κ such that there is an increasing chain $\{A_\alpha : \alpha < \kappa\} \subset \mathcal{I}$ with $\bigcup_{\alpha<\kappa} A_\alpha \notin \mathcal{I}$. We investigate which set A of regular cardinals can be the additivity spectrum of certain ideals. Assume that $\mathcal{I} = \mathcal{B}$ or $\mathcal{I} = \mathcal{N}$, where $\mathcal{B}$ denotes the σ-ideal generated by the compact subsets of the Baire space ω^ω, and $\mathcal{N}$ is the ideal of the null sets. We show that if A is a non-empty progressive set of uncountable regular cardinals and $\mathrm{pcf}(A) = A$, then $\mathrm{ADD}(\mathcal{I}) = A$ in some c.c.c generic extension of the ground model. On the other hand, we also show that if A is a countable subset of $\mathrm{ADD}(\mathcal{I})$, then $\mathrm{pcf}(A) \subset \mathrm{ADD}(\mathcal{I})$. For countable sets these results give a full characterization of the additivity spectrum of $\mathcal{I}$: a non-empty countable set A of uncountable regular cardinals can be $\mathrm{ADD}(\mathcal{I})$ in some c.c.c generic extension iff $A = \mathrm{pcf}(A)$.

Keywords: cardinal invariants, reals, pcf theory, null sets, meager sets, Baire space, additivity
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