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On meager function spaces, network character and meager convergence in topological spaces

Comment.Math.Univ.Carolin. 52,2 (2011) 273 –281.

Abstract: For a non-isolated point x of a topological space X let $\mathrm{nw}_\chi(x)$ be the smallest cardinality of a family $\mathcal{N}$ of infinite subsets of X such that each neighborhood $O(x) \subset X$ of x contains a set $N \in \mathcal{N}$. We prove that

- each infinite compact Hausdorff space X contains a non-isolated point x with $\mathrm{nw}_\chi(x) = \aleph_0$;
- for each point $x \in X$ with $\mathrm{nw}_\chi(x) = \aleph_0$ there is an injective sequence $(x_n)_{n\in\omega}$ in X that $\mathcal{F}$-converges to x for some meager filter $\mathcal{F}$ on ω;
- if a functionally Hausdorff space X contains an $\mathcal{F}$-convergent injective sequence for some meager filter $\mathcal{F}$, then for every path-connected space Y that contains two non-empty open sets with disjoint closures, the function space $C_p(X,Y)$ is meager.

Also we investigate properties of filters $\mathcal{F}$ admitting an injective $\mathcal{F}$-convergent sequence in $\beta\omega$.

Keywords: network character, meager convergent sequence, meager filter, meager space, function space
AMS Subject Classification: Primary 54A20, 54C35; Secondary 54E52

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