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Some fixed point theorems and existence of weak solutions of Volterra integral equation under Henstock-Kurzweil-Pettis integrability

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Abstract: In this paper we examine the set of weakly continuous solutions for a Volterra integral equation in Henstock-Kurzweil-Pettis integrability settings. Our result extends those obtained in several kinds of integrability settings. Besides, we prove some new fixed point theorems for function spaces relative to the weak topology which are basic in our considerations and comprise the theory of differential and integral equations in Banach spaces.

Keywords: fixed point theorems, Henstock-Kurzweil-Pettis integral, Volterra equation, measure of weak noncompactness

AMS Subject Classification: 47H10, 28B05, 45D05, 45N05, 26A39

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