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Comparison game on Borel ideals

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Abstract: We propose and study a "classification" of Borel ideals based on a natural infinite game involving a pair of ideals. The game induces a pre-order $\sqsubseteq$ and the corresponding equivalence relation. The pre-order is well founded and "almost linear". We concentrate on F_σ and $F_{\sigma\delta}$ ideals. In particular, we show that all F_σ-ideals are $\sqsubseteq$-equivalent and form the least equivalence class. There is also a least class of non-F_σ Borel ideals, and there are at least two distinct classes of $F_{\sigma\delta}$ non-F_σ ideals.

Keywords: ideals on countable sets, comparison game, Tukey order, games on integers
AMS Subject Classification: 03E15, 03E05

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