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On AP spaces in concern with compact-like sets and submaximality

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Abstract: The definitions of AP and WAP were originated in categorical topology by A. Pultr and A. Tozzi, *Equationally closed subframes and representation of quotient spaces*, Cahiers Topologie Géom. Différentielle Catég. **34** (1993), no. 3, 167–183. In general, we have the implications: $T_2 \Rightarrow KC \Rightarrow US \Rightarrow T_1$, where KC is defined as the property that every compact subset is closed and US is defined as the property that every convergent sequence has at most one limit. And a space is called *submaximal* if every dense subset is open. In this paper, we prove that: (1) every AP T_1-space is US, (2) every nodec WAP T_1-space is submaximal, (3) every submaximal and collectionwise Hausdorff space is AP. We obtain that, as corollaries, (1) every countably compact (or compact or sequentially compact) AP T_1-space is Fréchet-Urysohn and US, which is a generalization of Hong's result in *On spaces in which compact-like sets are closed, and related spaces*, Commun. Korean Math. Soc. **22** (2007), no. 2, 297–303, (2) if a space is nodec and T_3, then submaximality, AP and WAP are equivalent. Finally, we prove, by giving several counterexamples, that (1) in the statement that every submaximal T_3-space is AP, the condition T_3 is necessary and (2) there is no implication between nodec and WAP.

Keywords: AP, WAP, door, submaximal, nodec, unique sequential limit
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