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Cellularity and the index of narrowness in topological groups

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Abstract: We study relations between the cellularity and index of narrowness in topological groups and their G_δ-modifications. We show, in particular, that the inequalities $\mathrm{in}((H)_\tau) \leq 2^{\tau\cdot\mathrm{in}(H)}$ and $c((H)_\tau) \leq 2^{2^{\tau\cdot\mathrm{in}(H)}}$ hold for every topological group H and every cardinal $\tau \geq \omega$, where $(H)_\tau$ denotes the underlying group H endowed with the G_τ-modification of the original topology of H and $\mathrm{in}(H)$ is the index of narrowness of the group H. Also, we find some bounds for the *complexity* of continuous real-valued functions f on an arbitrary ω-narrow group G understood as the minimum cardinal $\tau \geq \omega$ such that there exists a continuous homomorphism $\pi\colon G \to H$ onto a topological group H with $w(H) \leq \tau$ such that $\pi \prec f$. It is shown that this complexity is not greater than 2^{2^ω} and, if G is weakly Lindelöf (or 2^ω-*steady*), then it does not exceed 2^ω.

Keywords: cellularity, G_δ-modification, index of narrowness, ω-narrow, weakly Lindelöf, $\mathbb{R}$-factorizable, complexity of functions
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