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### *Study of a viscoelastic frictional contact problem with adhesion*

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**Abstract:** We consider a quasistatic frictional contact problem between a viscoelastic body with long memory and a deformable foundation. The contact is modelled with normal compliance in such a way that the penetration is limited and restricted to unilateral constraint. The adhesion between contact surfaces is taken into account and the evolution of the bonding field is described by a first order differential equation. We derive a variational formulation and prove the existence and uniqueness result of the weak solution under a certain condition on the coefficient of friction. The proof is based on time-dependent variational inequalities, differential equations and Banach fixed point theorem.

**Keywords:** viscoelastic, normal compliance, adhesion, frictional, variational inequality, weak solution

**AMS Subject Classification:** 47J20, 49J40, 74M10, 74M15

#### REFERENCES

- [1] Andersson L.-E., *Existence result for quasistatic contact problem with Coulomb friction*, Appl. Math. Optim. **42** (2000), 169–202.
- [2] Bonetti E., Bonfanti G., Rossi R., *Global existence for a contact problem with adhesion*, Math. Methods Appl. Sci. **31** (2008), 1029–1064.
- [3] Bonetti E., Bonfanti G., Rossi R., *Well-posedness and long-time behaviour for a model of contact with adhesion*, Indiana Univ. Math. J. **56** (2007), 2787–2820.
- [4] Bonetti E., Bonfanti G., Rossi R., *Thermal effects in adhesive contact: Modelling and analysis*, Nonlinearity **22** (2009), 2697–2731.
- [5] Bonetti E., Bonfanti G., Rossi R., *Long-time behaviour of a thermomechanical model for adhesive contact* (preprint arXiv:0909.2493), Discrete Contin. Dyn. Syst. Ser. S, in print (2010).
- [6] Cangémi L., *Frottement et adhérence: modèle, traitement numérique application à l'interface fibre/matrice*, Ph.D. Thesis, Univ. Méditerranée, Aix Marseille I, 1997.
- [7] Chau O., Fernandez J.R., Shillor M., Sofonea M., *Variational and numerical analysis of a quasistatic viscoelastic contact problem with adhesion*, J. Comput. Appl. Math. **159** (2003), 431–465.
- [8] Chau O., Shillor M., Sofonea M., *Dynamic frictionless contact with adhesion*, J. Appl. Math. Phys. **55** (2004), 32–47.
- [9] Cocou M., Rocca R., *Existence results for unilateral quasistatic contact problems with friction and adhesion*, Math. Model. Num. Anal. **34** (2000), 981–1001.
- [10] Cocou M., Schryve M., Raous M., *A dynamic unilateral contact problem with adhesion and friction in viscoelasticity*, Z. Angew. Math. Phys. **61** (2010), 721–743.
- [11] Duvaut G., Lions J.-L., *Les inéquations en mécanique et en physique*, Dunod, Paris, 1972.
- [12] Eck C., Jarušek J., Krbeč M., *Unilateral Contact Problems. Variational Methods and Existence Theorems*, Pure Appl. Math., 270, Chapman & Hall / CRC, Boca Raton, 2005.
- [13] Fernandez J.R., Shillor M., Sofonea M., *Analysis and numerical simulations of a dynamic contact problem with adhesion*, Math. Comput. Modelling **37** (2003), 1317–1333.
- [14] Frémond M., *Adhérence des solides*, J. Méc. Théor. Appl. **6** (1987), 383–407.
- [15] Frémond M., *Equilibre des structures qui adhèrent à leur support*, C.R. Acad. Sci. Paris Sér. II **295** (1982), 913–916.
- [16] Frémond M., *Non-smooth Thermomechanics*, Springer, Berlin, 2002.
- [17] Hild P., *An example of nonuniqueness for the continuous static unilateral contact model with Coulomb friction*, C.R. Math. Acad. Sci. Paris **337** (2003), 685–688.
- [18] Jarušek J., Sofonea M., *On the solvability of dynamic elastic-visco-plastic contact problems*, Z. Angew. Math. Mech. **88** (2008), 3–22.
- [19] Jarušek J., Sofonea M., *On the solvability of dynamic elastic-visco-plastic contact problems with adhesion*, Ann. Acad. Rom. Sci. Ser. Math. Appl. **1** (2009), 191–214.

- [20] Kočvara M., Mielke A., Roubíček T., *A rate-independent approach to the delamination problem*, Math. Mech. Solids **11** (2006), 423–447.
- [21] Nassar S.A., Andrews T., Kruk S., Shillor M., *Modelling and simulations of a bonded rod*, Math. Comput. Modelling **42** (2005), 553–572.
- [22] Point N., *Unilateral contact with adherence*, Math. Methods Appl. Sci. **10** (1988), 367–381.
- [23] Raous M., Cangémi L., Cocu M., *A consistent model coupling adhesion, friction, and unilateral contact*, Comput. Methods Appl. Mech. Engrg. **177** (1999), 383–399.
- [24] Rocca R., *Analyse et numérique de problèmes quasi-statiques de contact avec frottement local de Coulomb en élasticité*, Thesis, Univ. Aix. Marseille 1, 2005.
- [25] Rojek J., Telega J.J., *Contact problems with friction, adhesion and wear in orthopedic biomechanics I: General developments*, J. Theor. Appl. Mech. **39** (2001), 655–677.
- [26] Rossi R., Roubíček T., *Thermodynamics and analysis of rate-independent adhesive contact at small strains*, preprint arXiv:1004.3764 (2010).
- [27] Roubíček T., L. Scardia L., C. Zanini C., *Quasistatic delamination problem*, Cont. Mech. Thermodynam. **21** (2009), 223–235.
- [28] Shillor M., Sofonea M., Telega J.J., *Models and Variational Analysis of Quasistatic Contact*, Lecture Notes in Physics, 655, Springer, Berlin, 2004.
- [29] Sofonea M., Han H., Shillor M., *Analysis and Approximations of Contact Problems with Adhesion or Damage*, Pure and Applied Mathematics, 276, Chapman & Hall / CRC Press, Boca Raton, Florida, 2006.
- [30] Sofonea M., Hoarau-Mantel T.V., *Elastic frictionless contact problems with adhesion*, Adv. Math. Sci. Appl. **15** (2005), 49–68.
- [31] Sofonea M., Matei A., *Variational inequalities with applications*, Advances in Mathematics and Mechanics, 18, Springer, New York, 2009.
- [32] Touzaline A., *Frictionless contact problem with adhesion for nonlinear elastic materials*, Electron. J. Differential Equations 2007, no. 174, 13 pp.
- [33] Touzaline A., *Frictionless contact problem with adhesion and finite penetration for elastic materials*, Ann. Pol. Math. **98** (2010), no. 1, 23–38.