Tejinder S. Neelon

On Boman's theorem on partial regularity of mappings

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Abstract: Let $\Lambda \subset \mathbb{R}^n \times \mathbb{R}^m$ and k be a positive integer. Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a locally bounded map such that for each $(\xi, \eta) \in \Lambda$, the derivatives $D^j_\xi f(x) := \frac{d^j}{dt^j} f(x + t\xi)\Big|_{t=0}$, $j = 1, 2, \dots k$, exist and are continuous. In order to conclude that any such map f is necessarily of class C^k it is necessary and sufficient that Λ be *not* contained in the zero-set of a nonzero homogenous polynomial $\Phi(\xi, \eta)$ which is linear in $\eta = (\eta_1, \eta_2, \dots, \eta_m)$ and homogeneous of degree k in $\xi = (\xi_1, \xi_2, \dots, \xi_n)$. This generalizes a result of J. Boman for the case $k = 1$. The statement and the proof of a theorem of Boman for the case $k = \infty$ is also extended to include the Carleman classes $C\{M_k\}$ and the Beurling classes $C(M_k)$ (Boman J., *Partial regularity of mappings between Euclidean spaces*, Acta Math. **119** (1967), 1–25).

Keywords: C^k maps, partial regularity, Carleman classes, Beurling classes
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