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Hyperplane section $\mathbb{OP}^2_0$ *of the complex Cayley plane as the homogeneous space* $\mathrm{F}_4/\mathrm{P}_4$

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Abstract: We prove that the exceptional complex Lie group F_4 has a transitive action on the hyperplane section of the complex Cayley plane $\mathbb{OP}^2$. Although the result itself is not new, our proof is elementary and constructive. We use an explicit realization of the vector and spin actions of $\mathrm{Spin}(9,\mathbb{C}) \leq \mathrm{F}_4$. Moreover, we identify the stabilizer of the F_4-action as a parabolic subgroup P_4 (with Levi factor $\mathrm{B}_3\mathrm{T}_1$) of the complex Lie group F_4. In the real case we obtain an analogous realization of ${\mathrm{F}_4}^{(-20)}/\P$.

Keywords: Cayley plane, octonionic contact structure, twistor fibration, parabolic geometry, Severi varieties, hyperplane section, exceptional geometry
AMS Subject Classification: Primary 32M12; Secondary 14M17

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