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Singular points of order k of Clarke regular and arbitrary functions

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Abstract: Let X be a separable Banach space and f a locally Lipschitz real function on X. For $k \in \mathbb{N}$, let $\Sigma_k(f)$ be the set of points $x \in X$, at which the Clarke subdifferential $\partial^C f(x)$ is at least k-dimensional. It is well-known that if f is convex or semiconvex (semiconcave), then $\Sigma_k(f)$ can be covered by countably many Lipschitz surfaces of codimension k. We show that this result holds even for each Clarke regular function (and so also for each approximately convex function). Motivated by a resent result of A.D. Ioffe, we prove also two results on arbitrary functions, which work with Hadamard directional derivatives and can be considered as generalizations of our theorem on $\Sigma_k(f)$ of Clarke regular functions (since each of them easily implies this theorem).

Keywords: Clarke regular functions, singularities, Hadamard derivative
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