Robert Černý

Sharp constants for Moser-type inequalities concerning embeddings into Zygmund spaces

Comment.Math.Univ.Carolin. 53,4 (2012) 557 –571.

Abstract: Let $n \geq 2$ and $\Omega \subset \mathbb{R}^n$ be a bounded set. We give a Moser-type inequality for an embedding of the Orlicz-Sobolev space $W_0 L^{\Phi}(\Omega)$, where the Young function Φ behaves like $t^n \log^{\alpha}(t)$, $\alpha < n-1$, for t large, into the Zygmund space $Z_0^{\frac{n-1-\alpha}{n}}(\Omega)$. We also study the same problem for the embedding of the generalized Lorentz-Sobolev space $W_0^m L^{\frac{n}{m},q} \log^{\alpha} L(\Omega)$, $m < n$, $q \in (1, \infty]$, $\alpha < \frac{1}{q'}$, embedded into the Zygmund space $Z_0^{\frac{1}{q'}-\alpha}(\Omega)$.

Keywords: Orlicz-Sobolev spaces, Lorentz-Sobolev spaces, Trudinger embedding, Moser-Trudinger inequality, best constants
AMS Subject Classification: 46E35, 46E30, 26D910

References

[1] Adachi S., Tanaka K., *Trudinger type inequalities in $\mathbb{R}^N$ and their best exponents*, Proc. Amer. Math. Soc. **128** (199), no. 7, 2051–2057.

[2] Adams D.R., *A sharp inequality of J. Moser for higher order derivatives*, Ann. of Math. **128** (1988), 385–398.

[3] Adimurthi, *Existence of positive solutions of the semilinear Dirichlet problem with critical growth for the n-Laplacian*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **17** (1990), 393–413.

[4] Adimurthi, *Positive solutions of the semilinear Dirichlet problem with critical growth in the unit disc in $\mathbb{R}^2$*, Proc. Indian Acad. Sci. Math. Sci. **99** (1989), 49–73.

[5] Alberico A., *Moser type inequalities for higher-order derivatives in Lorentz spaces*, Potential Anal. **28** (2008), 389–400.

[6] Alvino A. *A limit case of the Sobolev inequality in Lorentz spaces*, Rend. Accad. Sci. Fis. Mat. Napoli (4) **44** (1977), 105–112.

[7] Alvino A., Ferone V., Trombetti G., *Moser-type inequalities in Lorentz spaces*, Potential Anal. **5** (1996), 273–299.

[8] Cassani D., Ruf B., Tarsi C., *Best constants for Moser type inequalities in Zygmund spaces*, Mat. Contemp. **36** (2009), 79–90.

[9] Černý R., *Concentration-compactness principle for embedding into multiple exponential spaces*, Math. Inequal. Appl. **15** (2012), no. 1, 165–198.

[10] Černý R., Cianchi A., Hencl S., *Concentration-compactness principle for Moser-Trudinger inequalities: new results and proofs*, Ann. Mat. Pura Appl., to appear (preprint is available at `http://www.karlin.mff.cuni.cz/kma-preprints/`)

[11] Černý R., Gurka P., *Moser-type inequalities for generalized Lorentz-Sobolev spaces*, Houston. Math. J., to appear (preprint is available at `http://www.karlin.mff.cuni.cz/kma-preprints/`)

[12] Černý R., Mašková S., *A sharp form of an embedding into multiple exponential spaces*, Czechoslovak Math. J. **60** (2010), no. 3, 751–782.

[13] Cianchi A., *A sharp embedding theorem for Orlicz-Sobolev spaces*, Indiana Univ. Math. J. **45** (1996), 39–65.

[14] Cianchi A., *Moser-Trudinger inequalities without boundary conditions and isoperimetric problems*, Indiana Univ. Math. J. **54** (2005), 669–705.

[15] Edmunds D.E., Gurka P., Opic B., *Double exponential integrability of convolution operators in generalized Lorentz-Zygmund spaces*, Indiana Univ. Math. J. **44** (1995), 19–43.

[16] Edmunds D.E., Gurka P., Opic B., *Double exponential integrability, Bessel potentials and embedding theorems*, Studia Math. **115** (1995), 151–181.

[17] Edmunds D.E., Gurka P., Opic B., *Sharpness of embeddings in logarithmic Bessel-potential spaces*, Proc. Roy. Soc. Edinburgh Sect. A **126** (1996), 995-1009.

[18] Edmunds D.E., Gurka P., Opic B., *On embeddings of logarithmic Bessel potential spaces*, J. Funct. Anal. **146** (1997), 116–150.

[19] Edmunds D.E., Gurka P., Opic B., *Optimal Sobolev imbeddings involving rearrangement-invariant quasinorms*, J. Funct. Anal. **170** (2000), 307–355.

[20] Edmunds D.E., Krbec M., *Two limiting cases of Sobolev imbeddings*, Houston J. Math. **21** (1995), 119–128.

[21] Fusco N., Lions P.-L., Sbordone C., *Sobolev imbedding theorems in borderline cases*, Proc. Amer. Math. Soc. **124** (1996), 561–565.

[22] Hencl S., *A sharp form of an embedding into exponential and double exponential spaces*, J. Funct. Anal. **204** (2003), no. 1, 196–227.

[23] Lions P.L., *The concentration-compactness principle in the calculus of variations. The limit case, Part I*, Rev. Mat. Iberoamericana **1** (1985), no. 1, 145–201.

[24] Lorentz G.G., *On the theory of spaces* Λ, Pacific J. Math. **1** (1951), 411–429.

[25] Moser J., *A sharp form of an inequality by N. Trudinger*, Indiana Univ. Math. J. **20** (1971), 1077–1092.

[26] Opic B., Pick L., *On generalized Lorentz-Zygmund spaces*, Math. Inequal. Appl. **2** (1999), no. 3, 391–467.

[27] Pohozhaev S.I., *On the imbedding Sobolev theorem for* $pl = n$, Doklady Conference, Section Math., pp. 158–170, Moscow Power Inst., Moscow, 1965.

[28] Rao M.M., Ren Z.D., *Theory of Orlicz Spaces*, Monographs and Textbooks in Pure and Applied Mathematics, 146, Marcel Dekker, Inc., New York, 1991.

[29] Trudinger N.S., *On imbeddings into Orlicz spaces and some applications*, J. Math. Mech. **17** (1967), 473–484.

[30] Yudovich V.I., *Some estimates connected with integral operators and with solutions of elliptic equations*, Soviet Math. Doklady **2** (1961), 746–749.