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The two-parameter class of Schröder inversions

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Abstract: Infinite lower triangular matrices of generalized Schröder numbers are used to construct a two-parameter class of invertible sequence transformations. Their inverses are given by triangular matrices of coordination numbers. The two-parameter class of Schröder transformations is merged into a one-parameter class of stretched Riordan arrays, the left-inverses of which consist of matrices of crystal ball numbers. Schröder and inverse Schröder transforms of important sequences are calculated.

Keywords: generalized Schröder numbers, coordination numbers, crystal ball numbers, stretched Riordan array, triangular matrix, sequence transformation, inversion, left-inverse

AMS Subject Classification: Primary 05A10; Secondary 05A15, 05A19

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