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A generalization of Čech-complete spaces and Lindelöf Σ-spaces

Abstract: The class of s-spaces is studied in detail. It includes, in particular, all Čech-complete spaces, Lindelöf p-spaces, metrizable spaces with the weight $\leq 2^\omega$, but countable non-metrizable spaces and some metrizable spaces are not in it. It is shown that s-spaces are in a duality with Lindelöf Σ-spaces: X is an s-space if and only if some (every) remainder of X in a compactification is a Lindelöf Σ-space [Arhangel'skii A.V., *Remainders of metrizable and close to metrizable spaces*, Fund. Math. **220** (2013), 71–81]. A basic fact is established: the weight and the networkweight coincide for all s-spaces. This theorem generalizes the similar statement about Čech-complete spaces. We also study hereditarily s-spaces, provide various sufficient conditions for a space to be a hereditarily s-space, and establish that every metrizable space has a dense subspace which is a hereditarily s-space. It is also shown that every dense-in-itself compact hereditarily s-space is metrizable.

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