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A poset of topologies on the set of real numbers

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Abstract: On the set $\mathbb{R}$ of real numbers we consider a poset $\mathcal{P}_\tau(\mathbb{R})$ (by inclusion) of topologies $\tau(A)$, where $A \subseteq \mathbb{R}$, such that $A_1 \supseteq A_2$ iff $\tau(A_1) \subseteq \tau(A_2)$. The poset has the minimal element $\tau(\mathbb{R})$, the Euclidean topology, and the maximal element $\tau(\emptyset)$, the Sorgenfrey topology. We are interested when two topologies τ_1 and τ_2 (especially, for $\tau_2 = \tau(\emptyset)$) from the poset define homeomorphic spaces $(\mathbb{R}, \tau_1)$ and $(\mathbb{R}, \tau_2)$. In particular, we prove that for a closed subset A of $\mathbb{R}$ the space $(\mathbb{R}, \tau(A))$ is homeomorphic to the Sorgenfrey line $(\mathbb{R}, \tau(\emptyset))$ iff A is countable. We study also common properties of the spaces $(\mathbb{R}, \tau(A)), A \subseteq \mathbb{R}$.

Keywords: Sorgenfrey line, poset of topologies on the set of real numbers
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