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On the adaptive wavelet estimation of a multidimensional regression function under α -mixing dependence: Beyond the standard assumptions on the noise

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Abstract: We investigate the estimation of a multidimensional regression function f from n observations of an α -mixing process (Y, X) , where $Y = f(X) + \xi$, X represents the design and ξ the noise. We concentrate on wavelet methods. In most papers considering this problem, either the proposed wavelet estimator is not adaptive (i.e., it depends on the knowledge of the smoothness of f in its construction) or it is supposed that ξ is bounded or/and has a known distribution. In this paper, we go far beyond this classical framework. Under no boundedness assumption on ξ and no a priori knowledge on its distribution, we construct adaptive term-by-term thresholding wavelet estimators attaining “sharp” rates of convergence under the mean integrated squared error over a wide class of functions f .

Keywords: nonparametric regression; α -mixing dependence; adaptive estimation; wavelet methods; rates of convergence

AMS Subject Classification: 62G08, 62G20

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