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*When every flat ideal is projective*

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**Abstract:** In this paper, we study the class of rings in which every flat ideal is projective. We investigate the stability of this property under homomorphic image, and its transfer to various contexts of constructions such as direct products, and trivial ring extensions. Our results generate examples which enrich the current literature with new and original families of rings that satisfy this property.

**Keywords:** FP-ring; direct product; homomorphic image; amalgamation of rings;  $A \bowtie^f J$ ; trivial extension

**AMS Subject Classification:** 13D05, 13D02

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