

## Chuan Liu

### *Paratopological (topological) groups with certain networks*

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**Abstract:** In this paper, we discuss certain networks on paratopological (or topological) groups and give positive or negative answers to the questions in [13]. We also prove that a non-locally compact,  $k$ -gentle paratopological group is metrizable if its remainder (in the Hausdorff compactification) is a Fréchet-Urysohn space with a point-countable  $cs^*$ -network, which improves some theorems in [Liu C., *Metrizability of paratopological (semitopological) groups*, Topology Appl. **159** (2012), 1415–1420], [Liu C., Lin S., *Generalized metric spaces with algebraic structures*, Topology Appl. **157** (2010), 1966–1974].

**Keywords:** paratopological groups; topological groups; sequential neighborhood; networks; metrizable; compactifications; remainders

**AMS Subject Classification:** 54E20, 54E35, 54H11

#### REFERENCES

- [1] Arhangel'skiĭ A.V., *Mappings and spaces*, Russian Math. Surveys **21** (1966), 115–162.
- [2] Arhangel'skiĭ A.V., *More on remainders close to metrizable spaces*, Topology Appl. **154** (2007), 1084–1088.
- [3] Arhangel'skiĭ A.V., *Components of first-countability and various kinds of pseudoopen mappings*, Topology Appl. **158** (2011), 215–222.
- [4] Arhangel'skiĭ A.V., Choban M.M., *On remainders of rectifiable spaces*, Topology Appl. **157** (2010), 789–799.
- [5] Arhangel'skiĭ A.V., Okunev O.G., Pestov V.G., *Free topological groups over metrizable spaces*, Topology Appl. **33** (1989), 63–76.
- [6] Arhangel'skiĭ A.V., Tkachenko M., *Topological Groups and Related Structures*, Atlantis Press and World Sci., Hackensack, NJ, 2008.
- [7] Engelking R., *General Topology*, PWN, Polish Scientific Pub., Warszawa, 1977.
- [8] Franklin S., *Spaces in which sequences suffice*, Fund. Math. **57** (1965), 107–115.
- [9] Gruenhage G.,  *$k$ -spaces and products of closed images of metric spaces*, Proc. Amer. Math. Soc. **80** (1980), 478–482.
- [10] Gruenhage G., *Generalized metric spaces*, K. Kunen, J.E. Vaughan, eds., Handbook of Set-Theoretic Topology, North-Holland, 1984, pp. 423–501.
- [11] Gruenhage G., Michael E., Tanaka Y., *Spaces determined by point-countable covers*, Pacific J. Math. **113** (1984), no. 2, 303–332.
- [12] Lin S., *On sequence-covering  $s$ -maps*, Math. Adv. (China) **25** (1996), 548–551.
- [13] Lin F., *A note on paratopological groups with countable networks of sequential neighborhoods*, Topology Proc. **41** (2013), 9–16.
- [14] Liu C., *A note on paratopological groups*, Comment. Math. Univ. Carolin. **47** (2006), no. 4, 633–640.
- [15] Liu C., *Metrizability of paratopological (semitopological) groups*, Topology Appl. **159** (2012), 1415–1420.
- [16] Liu C., Lin S., *Generalized metric spaces with algebraic structures*, Topology Appl. **157** (2010), 1966–1974.
- [17] Michael E., *A quintuple quotient quest*, General Topology Appl. **2** (1972), 91–138.
- [18] Ordman E., Smith-Thomas B., *Sequential conditions and free topological groups*, Proc. Amer. Math. Soc. **79** (1980), no. 2, 319–326.
- [19] Simon P., *Divergent sequences in compact Hausdorff spaces*, Topology, Vol. II (Proc. Fourth Colloq., Budapest, 1978), pp. 1087–1094, Colloq. Math. Soc. János Bolyai, 23, North-Holland, Amsterdam-New York, 1980.
- [20] Shiraki T.,  *$M$ -spaces, their generalization and metrization theorems*, Sci. Rep. Tokyo Kyoiku Daigaku A, 11 (1971), 57–67.
- [21] Tanaka Y., *Point-countable covers and  $k$ -networks*, Topology Proc. **12** (1987), 327–349.