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An overview of free nilpotent Lie algebras

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Abstract: Any nilpotent Lie algebra is a quotient of a free nilpotent Lie algebra of the same nilindex and type. In this paper we review some nice features of the class of free nilpotent Lie algebras. We will focus on the survey of Lie algebras of derivations and groups of automorphisms of this class of algebras. Three research projects on nilpotent Lie algebras will be mentioned.

Keywords: Lie algebra; Levi subalgebra; nilpotent; free nilpotent; derivation; automorphism; representation
AMS Subject Classification: Primary 17B10; Secondary 17B30

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