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Positive solutions for a system of third-order differential equation with multi-point and integral conditions

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Abstract: This paper concerns the following system of nonlinear third-order boundary value problem:

$$u_i'''(t) + f_i(t, u_1(t), \dots, u_n(t), u_1'(t), \dots, u_n'(t)) = 0,\ 0 < t < 1,\ i \in \{1, \dots, n\}$$

with the following multi-point and integral boundary conditions:

$$\Big\{ u_i(0) = 0 u_i'(0) = 0 u_i'(1) = \sum_{j=1}^{p} \beta_{j,i} u_i'(\eta_{j,i}) + \int_0^1 h_i(u_1(s), \dots, u_n(s))\, ds$$

where $\beta_{j,i} > 0$, $0 < \eta_{1,i} < \cdots < \eta_{p,i} < \frac{1}{2}$, $f_i : [0,1] \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ and $h_i : [0,1] \times \mathbb{R}^n \to \mathbb{R}$ are continuous functions for all $i \in \{1, \dots, n\}$ and $j \in \{1, \dots, p\}$. Using Guo-Krasnosel'skii fixed point theorem in cone, we discuss the existence of positive solutions of this problem. We also prove nonexistence of positive solutions and we give some examples to illustrate our results.

Keywords: third-order differential equation; multi-point and integral boundary conditions; Guo-Krasnosel'skii fixed point theorem in cone; positive solutions
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References

[1] Gregus M., *Third order linear differential equations*, in Math. Appl., Reidel, Dordrecht, 1987.
[2] Yao Q., Feng Y., *The existence of solution for a third-order two-point boundary value problem*, Appl. Math. Lett. **15** (2002), 227–232.
[3] Feng Y., Liu S., *Solvability of a third-order two-point boundary value problem*, Appl. Math. Lett. **18** (2005), 1034–1040.
[4] Klaasen G., *Differential inequalities and existence theorems for second and third order boundary value problems*, J. Differential Equations **10** (1971), 529–537.
[5] Jackson L.K., *Existence and uniqueness of solutions of boundary value problems for third order differential equations*, J. Differential Equations **13** (1973), 432–437.
[6] O'Regan D., *Topological transversality: Application to third order boundary value problems*, SIAM J. Math. Anal. **19** (1987), 630–641.
[7] Sun Y., *Positive solutions for third-order three-point nonhomogeneous boundary value problems*, Appl. Math. Lett. **22** (2009) 45–51.
[8] Guo L.J., Sun J.P., Zhao Y.H., *Existence of positive solution for nonlinear third-order three-point boundary value problem*, Nonlinear Anal. **68** (2008), 3151–3158.
[9] Guo D., Lakshmikantham V., *Nonlinear Problems in Abstract Cones*, Academic Press, San Diego, 1988.
[10] Deimling K., *Nonlinear Functional Analysis*, Springer, Berlin, 1985.
[11] Zhang X.G., Liu L.S., and Wu C.X., *Nontrivial solution of third-order nonlinear eigenvalue problems*, Appl. Math. Comput. **176** (2006), 714–721.
[12] Chen H., *Positive solutions for the nonhomogeneous three-point boundary value problem of second order differential equations*, Math. Comput. Modelling **45** (2007), 844–852.
[13] Graef J.R., Yang B., *Multiple positive solution to a three-point third order boundary value problems*, Discrete Contin. Dyn. Syst. 2005, suppl., pp. 1–8.
[14] Ma R., *Positive solutions for a second order three-point boundary value problems*, Appl. Math. Lett. **14** (2001), 1–5.
[15] Yao Q.L., *The existence and multiplicity of positive solutions for third-order three-point boundary value problem*, Acta Math. Appl. Sinica **19** (2003), 117–122.
[16] Anderson D., *Multiple positive solutions for a three-point boundary value problem*, Math. Comput. Modelling **27** (1998), 49–57.
[17] Goodrich C.S., *Existence of a positive solution to a nonlocal semipositone boundary value problem on a time scale*, Comment. Math. Univ. Carolin. **54** (2013), no. 4, 509–525.

[18] Anderson D., *Green's function for a third-order generalized right focal problem*, J. Math. Anal. Appl. **288** (2003), 1–14.

[19] Boucherif A., Al-Malki N., *Nonlinear three-point third order boundary value problems*, Appl. Math. Comput. **190** (2007), 1168–1177.

[20] Kong L., Kong Q., *Multi-point boundary value problems of second-order differential equations* (I), Nonlinear. Anal. **58** (2004), 909–931.

[21] Kong L., Kong Q., *Multi-point boundary value problems of second-order differential equations* (II), Comm. Appl. Nonlinear. Anal. **14** (2007), 93–111.

[22] Henderson J., Ntouyas S.K., Purnaras I.K., *Positive solutions for systems of generalized three-point nonlinear boundary value problems*, Comment. Math. Univ. Carolin. **49** (2008), no. 1, 79–91.

[23] Jebari R., Boukricha A., *Solvability and positive solution of a system of second-order boundary value problem with integral boundary conditions*, Bound. Value Probl. 2014, DOI: 10.1186/s13661-014-0262-8.