Jan Starý
Coherent ultrafilters and nonhomogeneity

Abstract: We introduce the notion of a *coherent P-ultrafilter* on a complete ccc Boolean algebra, strengthening the notion of a *P*-point on ω, and show that these ultrafilters exist generically under $\mathfrak{c} = \mathfrak{d}$. This improves the known existence result of Ketonen [*On the existence of P-points in the Stone-Čech compactification of integers*, Fund. Math. **92** (1976), 91–94]. Similarly, the existence theorem of Canjar [*On the generic existence of special ultrafilters*, Proc. Amer. Math. Soc. **110** (1990), no. 1, 233–241] can be extended to show that *coherently selective ultrafilters* exist generically under $\mathfrak{c} = \mathrm{cov}\,\mathcal{M}$. We use these ultrafilters in a topological application: a coherent *P*-ultrafilter on an algebra $\mathcal{B}$ is an *untouchable point* in the Stone space of $\mathcal{B}$, witnessing its nonhomogeneity.

References

[BS]   Balcar B., Simon P., *On minimal π-character of points in extremally disconnected compact spaces*, Topology Appl. **41** (1991), 133–145.

[C]   Canjar R.M., *On the generic existence of special ultrafilters*, Proc. Amer. Math. Soc. **110** (1990), no. 1, 233–241.

[F]   Frolík Z., *Maps of extremally disconnected spaces, theory of types, and applications*, in Franklin, Frolík, Koutník (eds.), General Topology and Its Relations to Modern Analysis and Algebra, Proceedings of the Kanpur topological conference (1971), pp. 131–142.

[K]   Ketonen J., *On the existence of P-points in the Stone-Čech compactification of integers*, Fund. Math. **92** (1976), 91–94.

[S]   Simon P., *Points in extremally disconnected compact spaces*, Rend. Circ. Mat. Palermo (2). Suppl. 24 (1990), 203–213.

[W]   Wimmer E.L., *The Shelah P-point independence theorem*, Israel J. Math **43** (1982), no. 1, 28–48.